Неупорядоченная модель Хаббарда с притяжением

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План доклада

- БКШ - БЭК кроссовер
- Модель Хаббарда с притяжением и DMFT
- DMFT+Σ и модель Хаббарда с притяжением
- Расчет T_c в DMFT и метод Нозьера - Шмитт-Ринк
- Тс в неупорядоченной модели Хаббарда
- Разложение Гинзбурга - Ландау
- Обобщенная теорема Андерсона

БКШ – БЭК кроссовер

A. J. Leggett (Springer, Berlin 1980),
P. Nozieres and S. Schmitt-Rink, J. Low Temp. Phys. 59, 195 (1985)

Слабая связь (БКШ) → Очень сильная связь (БЭК)

- Рыхлые пары ($\xi >> a$)
- Куперовские пары образуются сразу в конденсате ($q=0$)
- $T_c \sim \Delta(0)$

- Компактные пары ($\xi < a$)
- Образуются при $T^* \sim \Delta(0) >> T_c$ не в конденсате
- T_c определяется условием БЭК

Модель Хаббарда с притяжением (DMFT)

$T_c \sim 2D\exp(-1/N(0)U)$

$T_c \sim t^2/U$

Неупорядоченная модель Хаббарда с притяжением

Будем рассматривать немагнитную модель Хаббарда с беспорядком. Гамильтониан рассматриваемой модели имеет вид:

$$H = -t \sum_{\langle ij \rangle \sigma} a^{\dagger}_{i\sigma} a_{j\sigma} + \sum_{i\sigma} \epsilon_i n_{i\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}, \tag{1}$$

где $t > 0$ – амплитуда перескока между ближайшими соседями, U – взаимодействие на узле (в дальнейшем, в данной работе мы будем интересоваться в основном случаем притяжения на узле, $U < 0$), $n_{i\sigma} = a^{\dagger}_{i\sigma} a_{i\sigma}$ – оператор числа электронов на узле, $a_{i\sigma}$ ($a^{\dagger}_{i\sigma}$) – оператор уничтожения (рождения) электрона со спином σ, локальные энергии ϵ_i полагаются независимыми случайными величинами на разных узлах решетки. Для упрощения диаграмной техники, в дальнейшем мы предполагаем гауссовское распределение для энергетических уровней ϵ_i:

$$\mathcal{P}(\epsilon_i) = \frac{1}{\sqrt{2\pi}\Delta} \exp\left(-\frac{\epsilon_i^2}{2\Delta^2}\right) \tag{2}$$

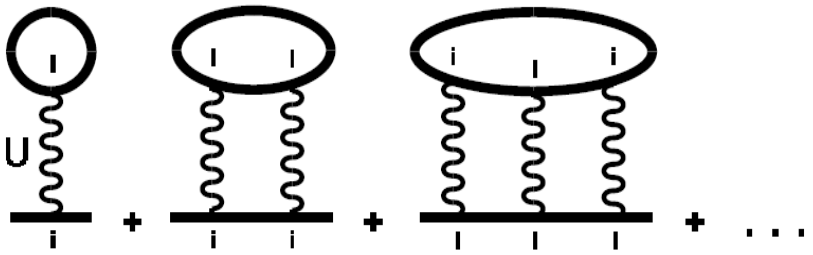
Параметр Δ здесь служит мерой силы беспорядка и гауссовское случайное поле ("белый" шум) энегетических уровней на различных узлах решетки вызывает "примесное" рассеяние, приводя к стандартной диаграммной технике для вычисления усредненных функций Грина[20].

Нормальная (несверхпроводящая) фаза

Traditional DMFT approach

$$H = -\sum_{ij,\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$$

J.Hubbard (1964)



In the limit of spatial dimensionality

$$d \to \infty$$

self - energy becomes ***local***, i.e.

$$\Sigma_{\mathbf{k}}(i\omega) \to \Sigma(i\omega)$$

Then the problem can be ***exactly*** (!) mapped on the equvalent problem of Anderson impurity, which is to be solved by some kind of "impurity solver"

W.Metzner, D.Vollhardt, 1989, A.Georges, G.Kotliar, 1992, Th.Pruschke, M.Jarrell, 1992

The absence of **k** - dependence of electron self - energy is the basic shortcoming of the traditional DMFT, however only due to this fact we obtain an exact solution!

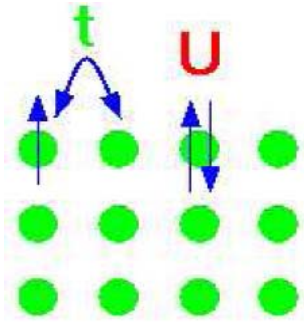
Standard DMFT calculation proceeds as follows:

1. Guess some initial value of *local* self – energy $\Sigma(i\omega)$, e.g. $\Sigma(i\omega) = 0$.

2. Calculate local Green's function as:

$$G_{ii}(i\omega) = \frac{1}{N}\sum_{\mathbf{k}} \frac{1}{i\omega + \mu - \varepsilon(\mathbf{k}) - \Sigma(i\omega)}$$

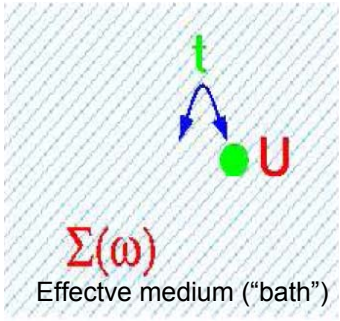
Hubbard



3. Define the "Weiss field" as:

$$\mathcal{G}_0^{-1}(i\omega) = \Sigma(i\omega) + G_{ii}^{-1}(i\omega)$$

Effective Anderson



4. Using some "impurity solver" calculate Green's function for the effective Anderson impurity, defined by Grassmanian functional integral:

$$G_d(\tau - \tau') = \frac{1}{Z_{\text{eff}}} \int Dc_{i\sigma}^{+} Dc_{i\sigma} c_{i\sigma}(\tau) c_{i\sigma}^{+}(\tau') \exp(-S_{\text{eff}})$$

with effective action for a fixed site ("impurity") i

$$S_{\text{eff}} = -\int_0^\beta d\tau_1 \int_0^\beta d\tau_2 c_{i\sigma}(\tau_1)\mathcal{G}_0^{-1}(\tau_1 - \tau_2)c_{i\sigma}^{+}(\tau_2) + \int_0^\beta d\tau U n_{i\uparrow}(\tau) n_{i\downarrow}(\tau)$$

and $Z_{\text{eff}} = \int Dc_{i\sigma}^{+} Dc_{i\sigma} \exp(-S_{\text{eff}})$, with $\beta = T^{-1}$, thus defining in fact the *new* value of $G_d^{-1}(i\omega)$.

5. Define the *new* value of local self – energy as:

$$\Sigma(i\omega) = \mathcal{G}_0^{-1}(i\omega) - G_d^{-1}(i\omega)$$

6. Using this new value as "initial" in step 1 continue the procedure until (and if) convergence is reached to obtain:

$$G_{ii}(i\omega) = G_d(i\omega)$$

Основы DMFT+Σ подхода

- **DMFT+Σ:**

M.V.Sadovskii, I.A.Nekrasov,
E.Z.Kuchinskii, Th.Pruschke,
V.I.Anisimov (2005)

$$G_{\mathbf{k}}(i\omega) = \frac{1}{i\omega + \mu - \varepsilon(\mathbf{k}) - \Sigma(i\omega) - \Sigma_{\mathbf{k}}(i\omega)}$$

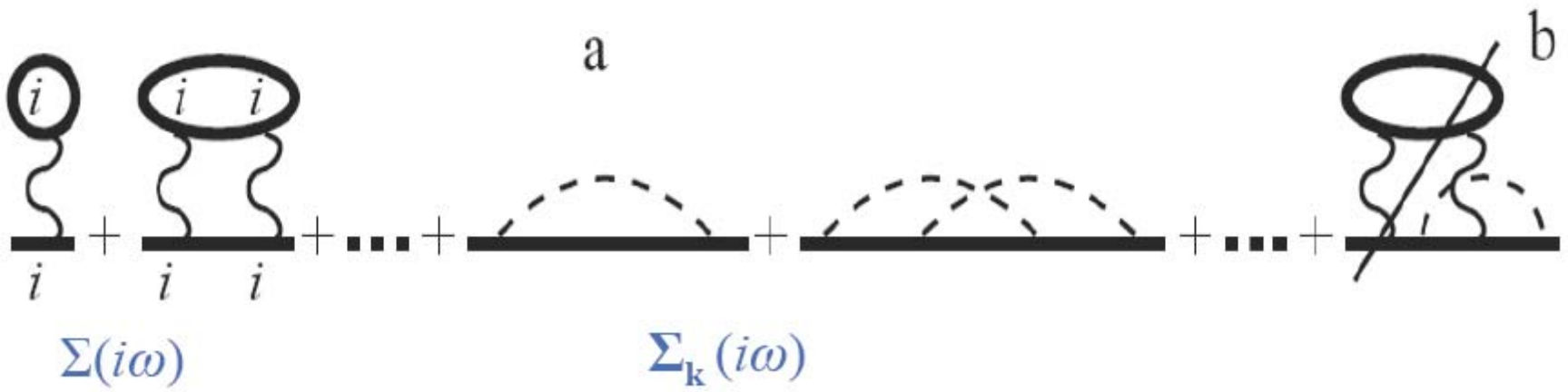
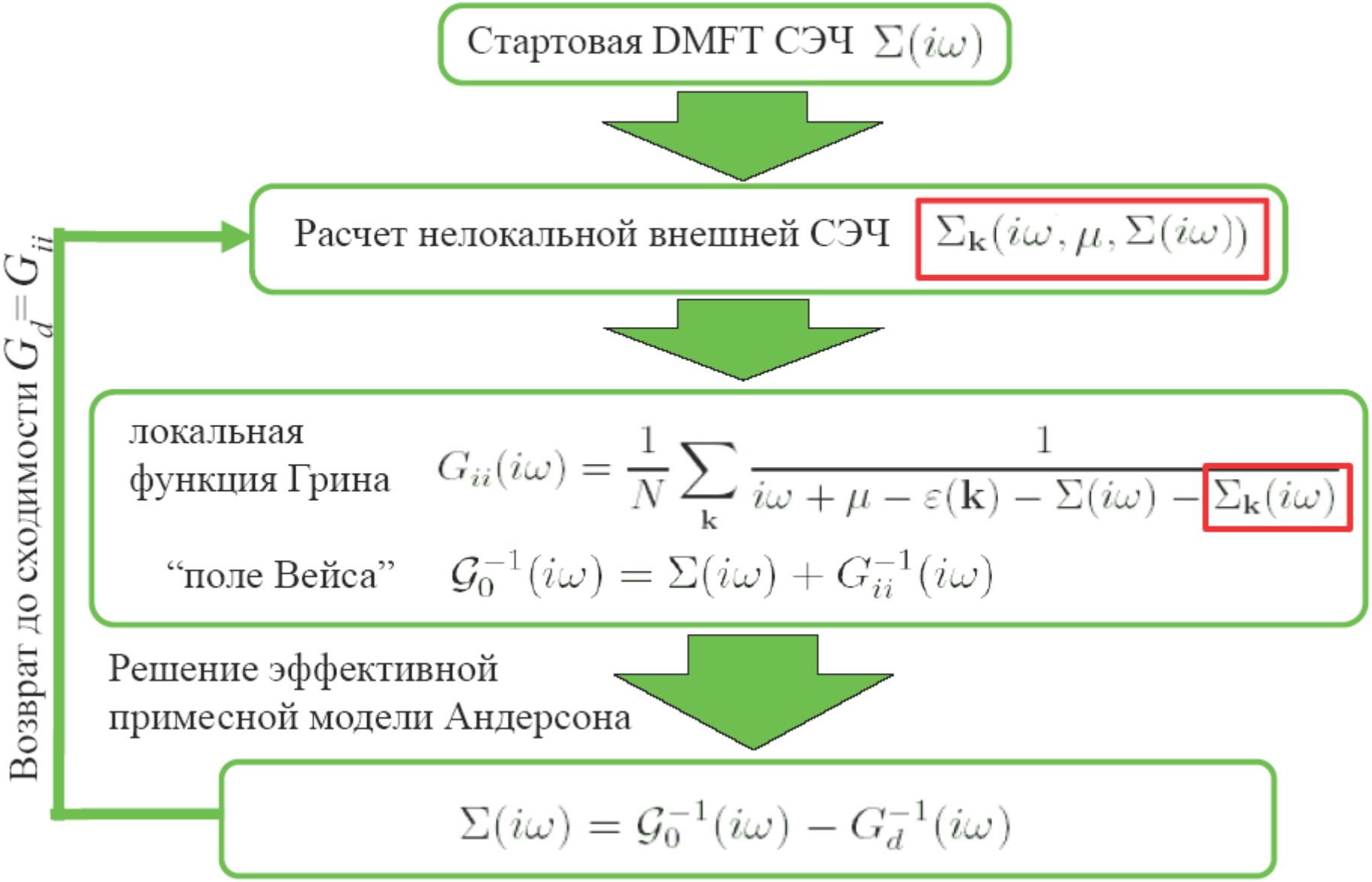


Схема уравнений DMFT+Σ_k подхода:



DMFT+Σ calculation scheme:

1. Guess some initial value of *local* self – energy $\Sigma(i\omega)$, e.g. $\Sigma(i\omega) = 0$.

2. Construct $\Sigma_{\mathbf{p}}(i\varepsilon)$ within some (approximate) scheme, taking into account interactions with "external" interaction (impurity scattering in our case) which in general can depend on $\Sigma(i\omega)$ and μ.

3. Calculate local Green's function as:

$$G_{ii}(i\omega) = \frac{1}{N}\sum_{\mathbf{k}} \frac{1}{i\omega + \mu - \varepsilon(\mathbf{k}) - \Sigma(i\omega) - \Sigma_{\mathbf{k}}(i\omega)}$$

4. Define the "Weiss field" as:

$$\mathcal{G}_0^{-1}(i\omega) = \Sigma(i\omega) + G_{ii}^{-1}(i\omega)$$

5. Using some "impurity solver" calculate Green's function for the effective Anderson impurity, defined by Grassmanian functional integral:

$$G_d(\tau - \tau') = \frac{1}{Z_{\text{eff}}} \int Dc^+_{i\sigma} Dc_{i\sigma} c_{i\sigma}(\tau) c^+_{i\sigma}(\tau') \exp(-S_{\text{eff}})$$

with effective action for a fixed site ("impurity") i

$$S_{\text{eff}} = -\int_0^\beta d\tau_1 \int_0^\beta d\tau_2 c_{i\sigma}(\tau_1) \mathcal{G}_0^{-1}(\tau_1 - \tau_2) c^+_{i\sigma}(\tau_2) + \int_0^\beta d\tau U n_{i\uparrow}(\tau) n_{i\downarrow}(\tau)$$

and $Z_{\text{eff}} = \int Dc^+_{i\sigma} Dc_{i\sigma} \exp(-S_{\text{eff}})$, with $\beta = T^{-1}$, thus defining in fact the *new* value of $G_d^{-1}(i\omega)$.

6. Define the *new* value of local self – energy as:

$$\Sigma(i\omega) = \mathcal{G}_0^{-1}(i\omega) - G_d^{-1}(i\omega)$$

7. Using this new value as "initial" in step 1 continue the procedure until (and if) convergence is reached to obtain:

$$\boxed{G_{ii}(i\omega) = G_d(i\omega)}$$

Σ для примесного рассеяния

Self-consistent Born approximation

$$\Sigma_{\mathbf{p}}(i\varepsilon) = \Delta^2 \sum_{\mathbf{p}} G(i\varepsilon, \mathbf{p}) \equiv \Sigma_{imp}(i\varepsilon)$$

$$\Sigma^{R,A}_{imp}(\varepsilon) = \Delta^2 \sum_{\mathbf{p}} G^{R,A}(\varepsilon, \mathbf{p}) = \mathrm{Re}\Sigma_{imp}(\varepsilon) \pm i\gamma(\varepsilon)$$

$$\gamma(\varepsilon) = \pi\Delta^2 N(\varepsilon)$$

Semi - elliptic DOS:

$$N(\varepsilon) = \frac{2}{\pi D^2}\sqrt{D^2 - \varepsilon^2} \qquad W = 2D \qquad W_{eff} = W\sqrt{1 + 16\frac{\Delta^2}{W^2}}$$

$$G_{ii} = \int_{-D}^{D} d\varepsilon' \frac{N_0(\varepsilon')}{\varepsilon + \mu - \varepsilon' - \Sigma(\varepsilon) - \Delta^2 G_{ii}} = \int_{-D}^{D} d\varepsilon' \frac{N_0(\varepsilon')}{E_t - \varepsilon'}, \tag{8}$$

where we have introduced the notation $E_t = \varepsilon + \mu - \Sigma(\varepsilon) - \Delta^2 G_{ii}$. In the case of semi – elliptic density of states this integral is easily calculated in analytic form, so that the local Green's function is written as:

$$G_{ii} = 2\frac{E_t - \sqrt{E_t^2 - D^2}}{D^2}. \tag{9}$$

It is easily seen that Eq. (9) represents one of the roots of quadratic equation:

$$G_{ii}^{-1} = E_t - \frac{D^2}{4} G_{ii}, \tag{10}$$

corresponding to the correct limit of $G_{ii} \to E_t^{-1}$ for infinitely narrow ($D \to 0$) band. Then

$$G_{ii}^{-1} = \varepsilon + \mu - \Sigma(\varepsilon) - \Delta^2 G_{ii} - \frac{D^2}{4} G_{ii} = \varepsilon + \mu - \Sigma(\varepsilon) - \frac{D_{eff}^2}{4} G_{ii}, \tag{11}$$

where we have introduced D_{eff} – an effective half-width of the band (in the absence of electronic correlations, i.e. for $U = 0$) widened by disorder scattering:

$$D_{eff} = D\sqrt{1 + 4\frac{\Delta^2}{D^2}}. \tag{12}$$

Eq. (10) was obtained from (8), thus comparing (11) and (10), we obtain:

$$G_{ii} = \int_{-D_{eff}}^{D_{eff}} d\varepsilon' \frac{\tilde{N}_0(\varepsilon')}{\varepsilon + \mu - \varepsilon' - \Sigma(\varepsilon)}, \tag{13}$$

Here

$$\tilde{N}_0(\varepsilon) = \frac{2}{\pi D_{eff}^2}\sqrt{D_{eff}^2 - \varepsilon^2} \tag{14}$$

Оптическая проводимость - DMFT+Σ

Э.З.Кучинский, И.А.Некрасов, М.В.Садовский (2006)

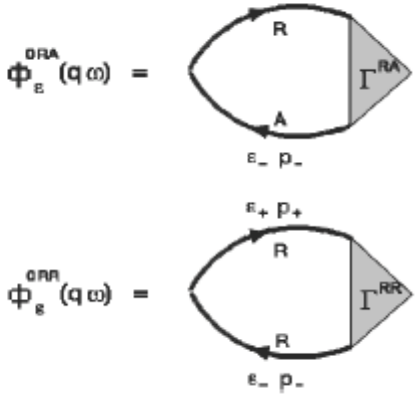
$$Re\sigma(\omega) = \frac{e^2\omega}{2\pi}\int_{-\infty}^{\infty} d\varepsilon\,[f(\varepsilon_-) - f(\varepsilon_+)]\,Re\left\{\phi_\varepsilon^{0RA}(\omega)\left[1 - \frac{\Sigma^R(\varepsilon_+) - \Sigma^A(\varepsilon_-)}{\omega}\right]^2 - \right.$$
$$\left. -\phi_\varepsilon^{0RR}(\omega)\left[1 - \frac{\Sigma^R(\varepsilon_+) - \Sigma^R(\varepsilon_-)}{\omega}\right]^2\right\}$$

$$\phi_\varepsilon^{0RA}(\omega) = \lim_{q\to 0}\frac{\Phi_\varepsilon^{0RA}(\omega,\mathbf{q}) - \Phi_\varepsilon^{0RA}(\omega,0)}{q^2}$$

$$\phi_\varepsilon^{0RR}(\omega) = \lim_{q\to 0}\frac{\Phi_\varepsilon^{0RR}(\omega,\mathbf{q}) - \Phi_\varepsilon^{0RR}(\omega,0)}{q^2}$$

$$\Phi_\varepsilon^{0RA}(\omega,\mathbf{q}) = \sum_{\mathbf{p}} G^R(\varepsilon_+,\mathbf{p}_+)G^A(\varepsilon_-,\mathbf{p}_-)\Gamma^{RA}(\varepsilon_-,\mathbf{p}_-;\varepsilon_+,\mathbf{p}_+)$$

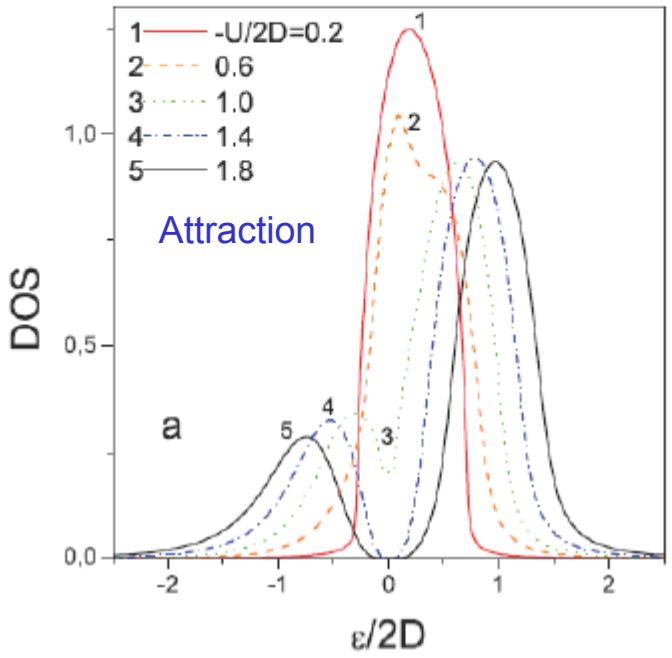
$$\Phi_\varepsilon^{0RR}(\omega,\mathbf{q}) = \sum_{\mathbf{p}} G^R(\varepsilon_+,\mathbf{p}_+)G^R(\varepsilon_-,\mathbf{p}_-)\Gamma^{RR}(\varepsilon_-,\mathbf{p}_-;\varepsilon_+,\mathbf{p}_+)$$

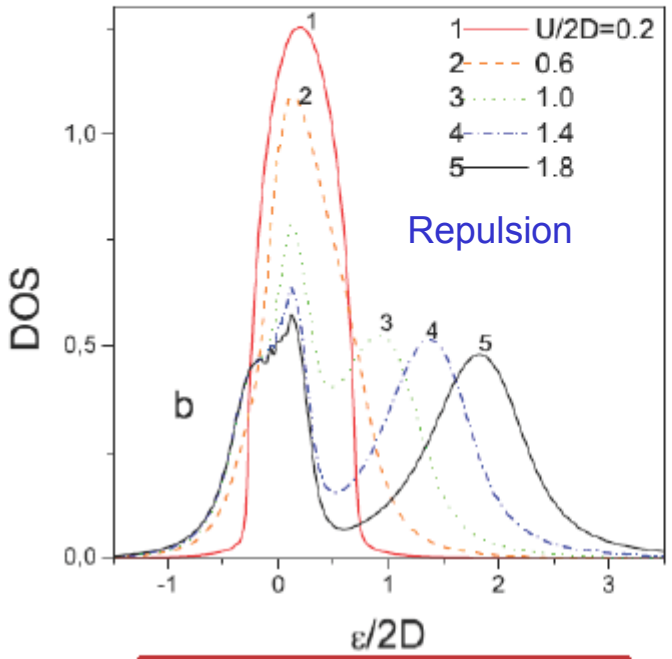


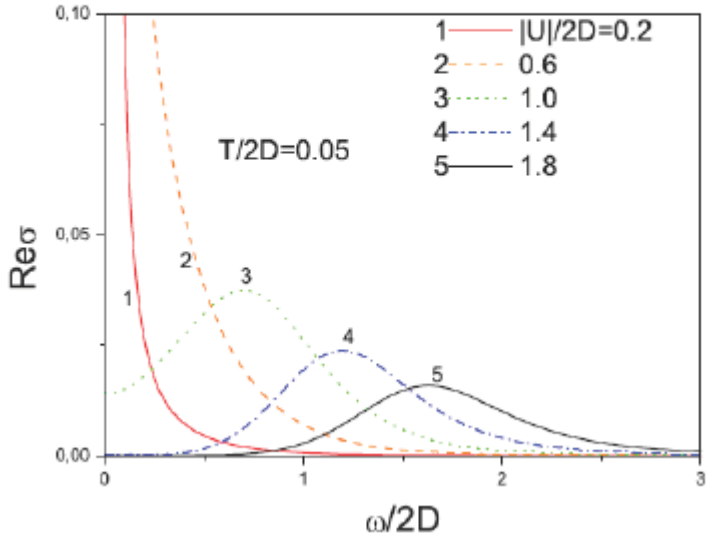
! Не содержат вершинных поправок от U

Находятся из обобщения самосогласованной теории локализации

Плотность состояний (Δ=0)







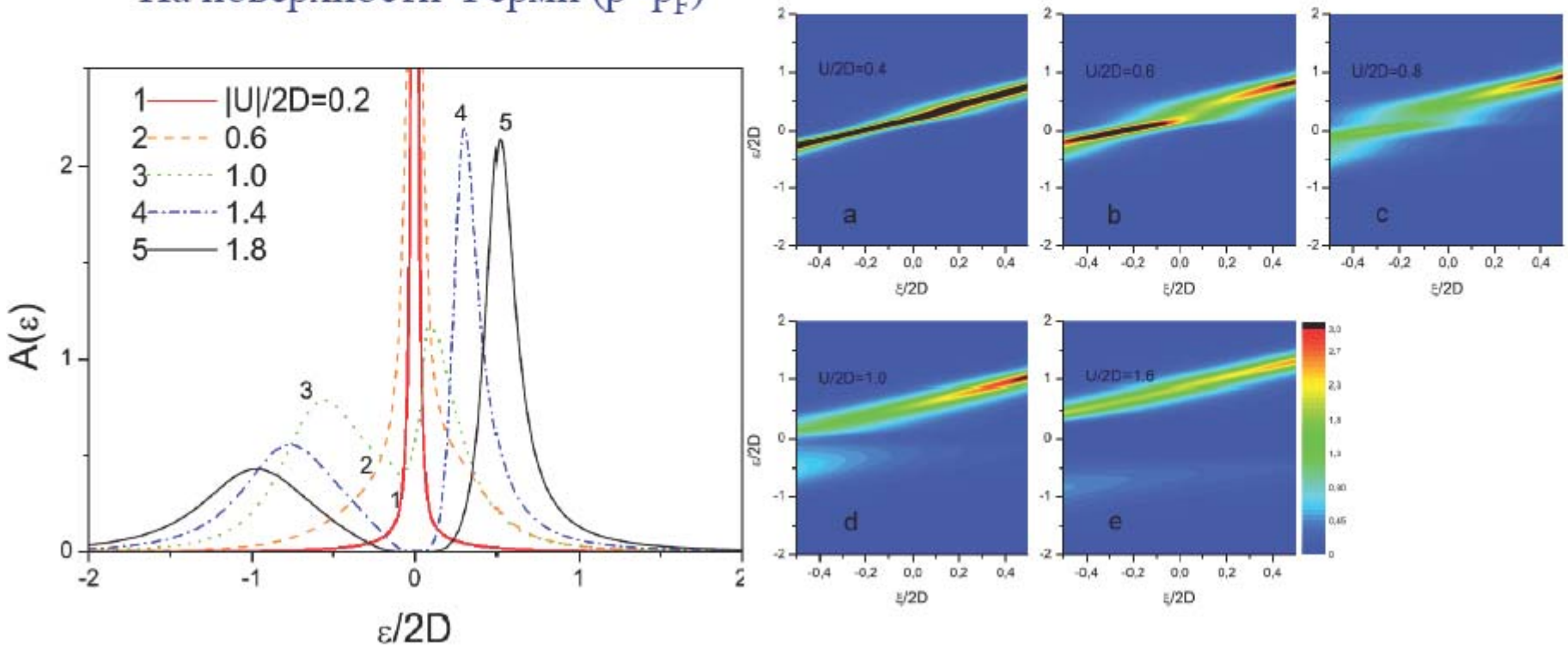
$n=<n_{\uparrow}>=0.25$

Оптическая проводимость

Спектральная плотность (Δ=0)

$$A(\varepsilon, \mathbf{p}) = -\frac{1}{\pi} Im G^R(\varepsilon, \mathbf{p})$$

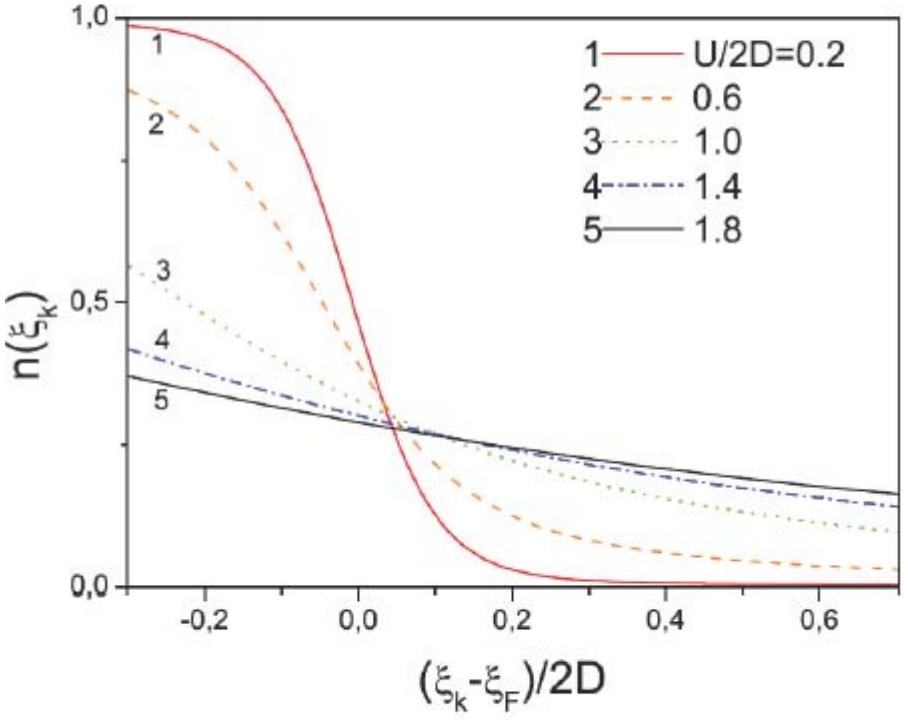
На поверхности Ферми ($p=p_F$)

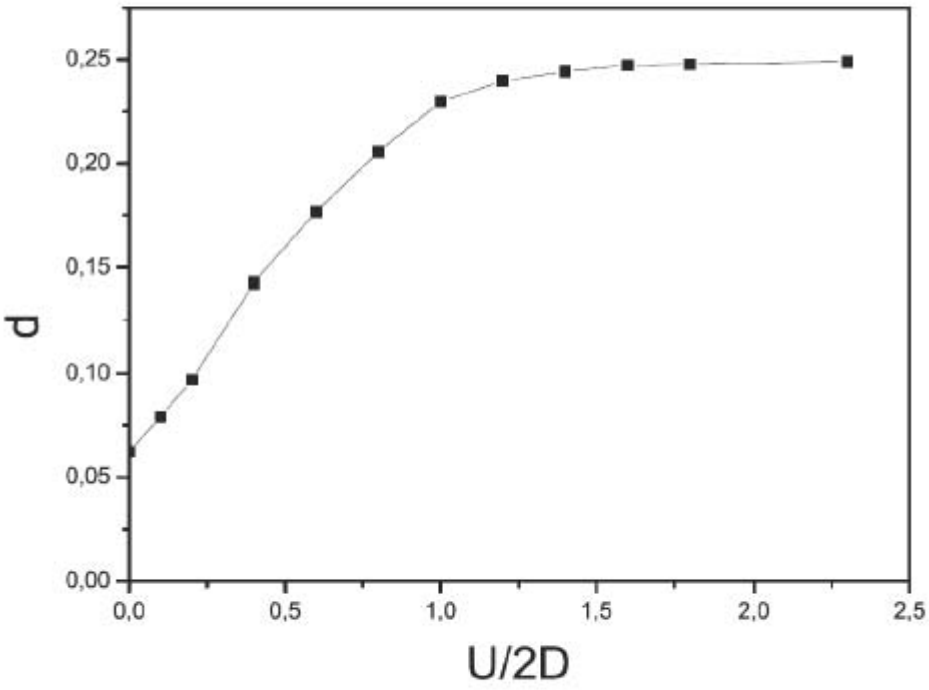


Функция распределения и число «двоек»

$$n(\xi_k) = \int_{-\infty}^{\infty} d\varepsilon A(\varepsilon, \xi_k) f(\varepsilon)$$

$d=\langle n_\uparrow n_\downarrow \rangle$

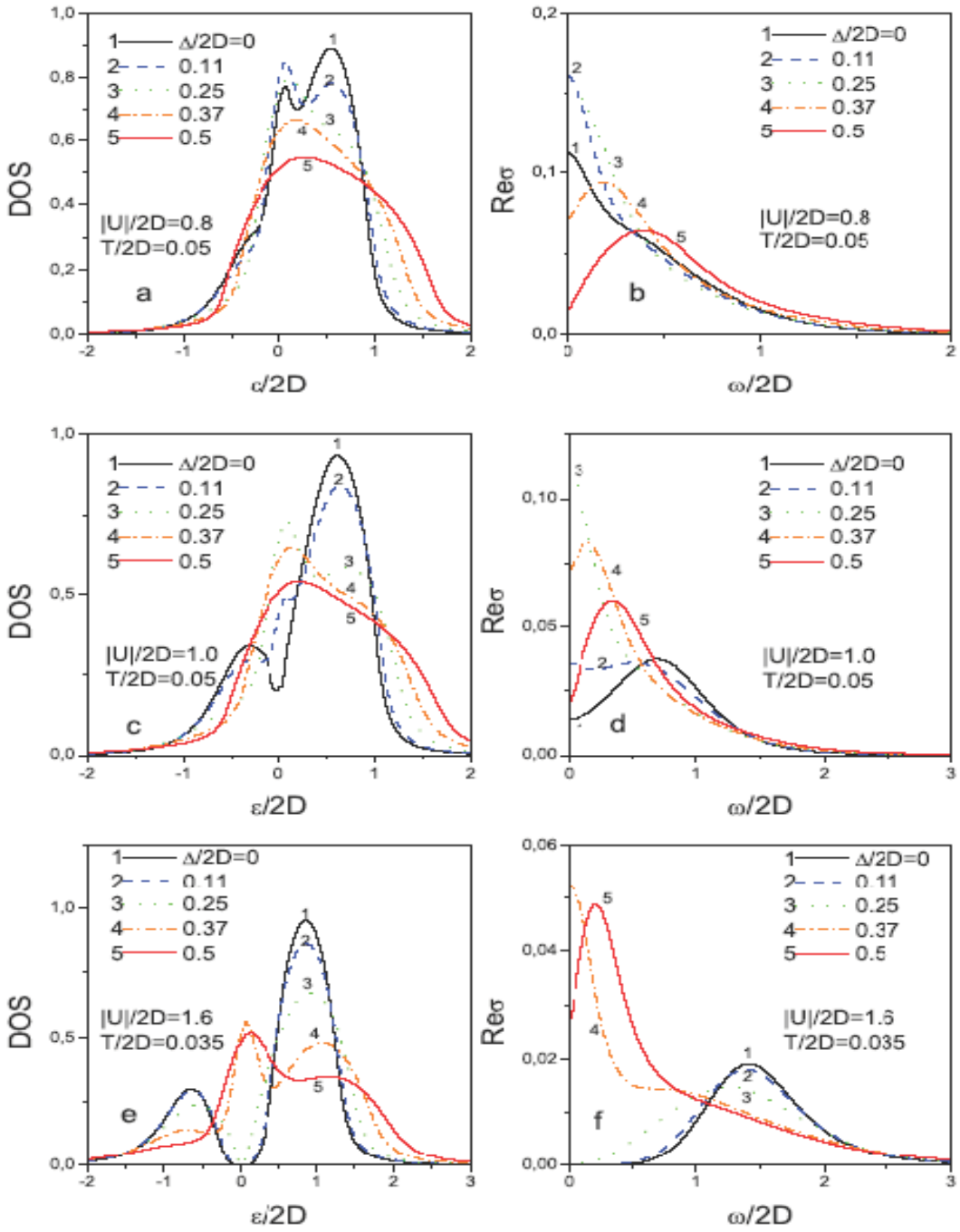




$n=\langle n_\uparrow \rangle=0.25$

$d \to n^2$ при $U/2D \ll 1$

$d \to n$ при $U/2D \gg 1$

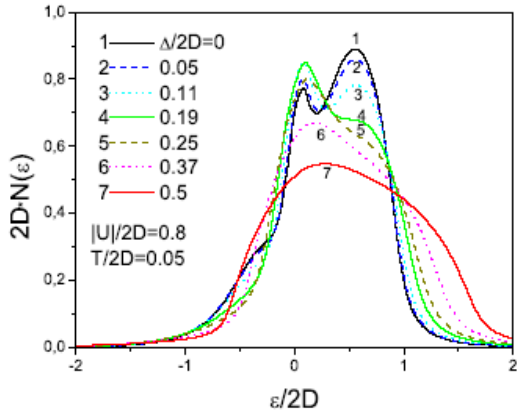


Влияние беспорядка

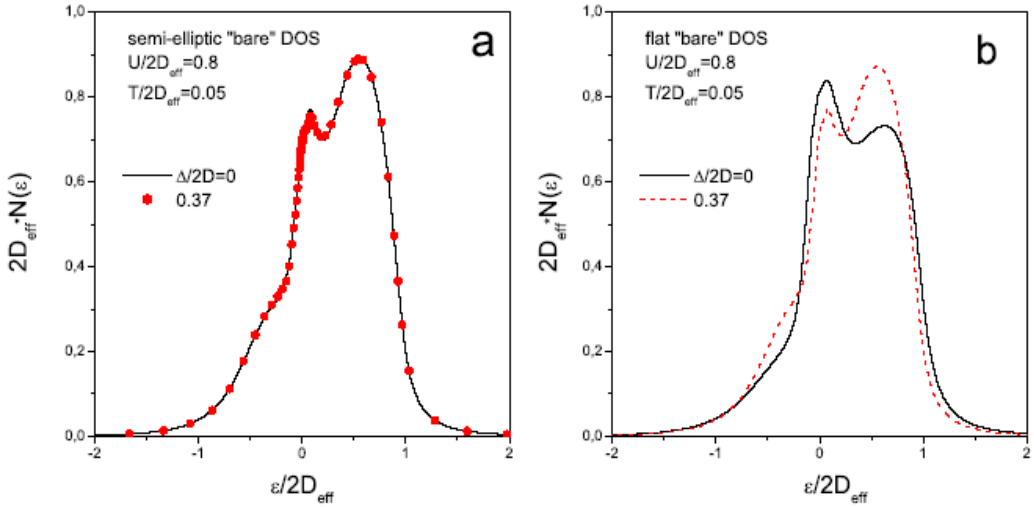
Плотность состояний и оптическая проводимость

Anderson transition at Δ=0.37 (T=0)

Universality of DOS dependence on disorder:

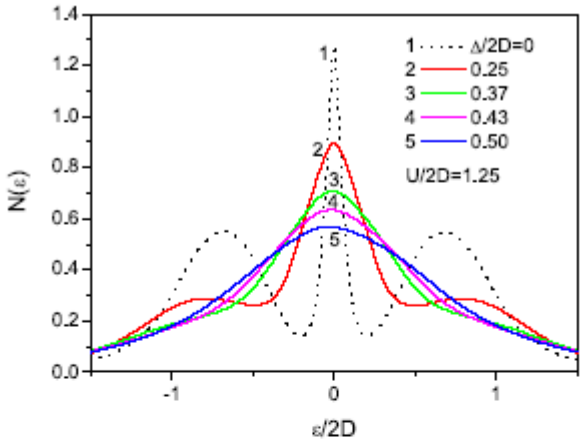


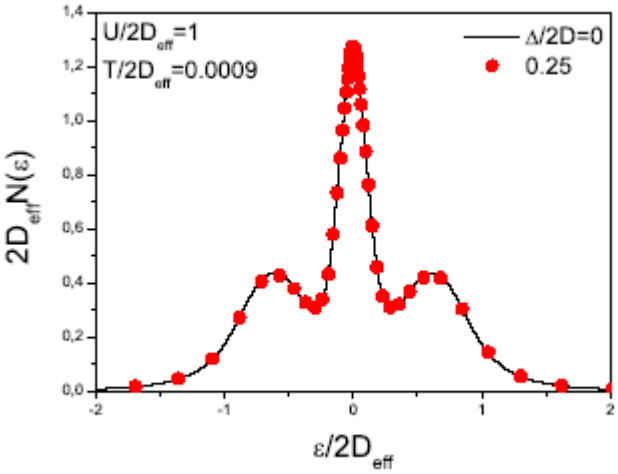
Dependence of the density of states on disorder in the model with semi – elliptic band.

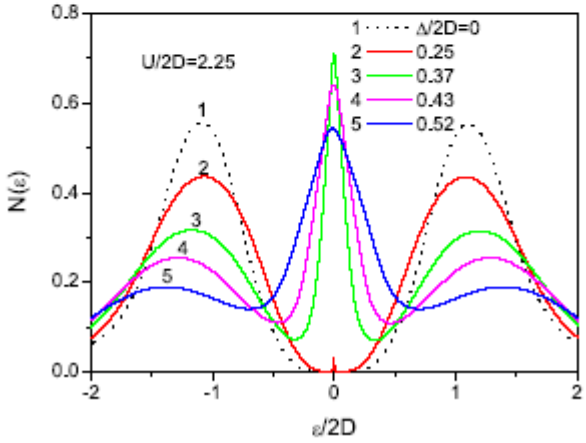


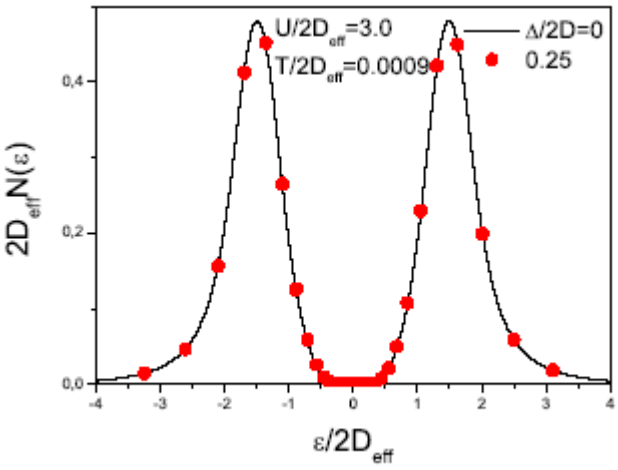
ɨ. 2: Universal dependence of the density of states on disorder: (a) — the model of semi – elliptic "bare" density of states; (b) — the model of flat "bare" density of states.

Universality of DOS dependence on disorder – repulsive Hubbard:









DISORDER INFLUENCE ON SUPERCONDUCTING TRANSITION

In general, Nozieres – Schmitt-Rink approach assumes, that corrections due to strong pairing attraction significantly change the chemical potential of the system, while possible correction due to this interaction to Cooper instability condition can be neglected, so that we can always use here the weak coupling (ladder) approximation. In such approximation the condition of Cooper instability in disordered Hubbard model takes the form:

$$1 = U\chi_0(q = 0, \omega_m = 0) \tag{15}$$

where

$$\chi_0(q = 0, \omega_m = 0) = T\sum_n \sum_{\mathbf{pp'}} \Phi_{\mathbf{pp'}}(\varepsilon_n) \tag{16}$$

represents the two – particle loop (susceptibility) in Cooper channel "dressed" only by disorder scattering, and $\Phi_{\mathbf{pp'}}(\varepsilon_n)$ is the averaged two – particle Green's function in Cooper channel ($\omega_m = 2\pi mT$ and $\varepsilon_n = \pi T(2n + 1)$ are the usual Boson and Fermion Matsubara frequencies).

To obtain $\sum_{\mathbf{pp'}} \Phi_{\mathbf{pp'}}(\varepsilon_n)$ we use the exact Ward identity

$$G(\varepsilon_n, \mathbf{p}) - G(-\varepsilon_n, -\mathbf{p}) = -\sum_{\mathbf{p'}} \Phi_{\mathbf{pp'}}(\varepsilon_n)(G_0^{-1}(\varepsilon_n, \mathbf{p'}) - G_0^{-1}(-\varepsilon_n, -\mathbf{p'})), \tag{17}$$

Here $G(\varepsilon_n, \mathbf{p})$ is the impurity averaged (but not containing Hubbard interaction corrections!) single – particle Green's function. Using the obvious symmetry $\varepsilon(\mathbf{p}) = \varepsilon(-\mathbf{p})$ and

$G(\varepsilon_n, -\mathbf{p}) = G(\varepsilon_n, \mathbf{p})$, we obtain from the Ward identity (17):

$$\sum_{\mathbf{pp'}} \Phi_{\mathbf{pp'}}(\varepsilon_n) = -\frac{\sum_{\mathbf{p}} G(\varepsilon_n, \mathbf{p}) - \sum_{\mathbf{p}} G(-\varepsilon_n, \mathbf{p})}{2i\varepsilon_n}, \tag{18}$$

so that for Cooper susceptibility (16) we have:

$$\chi_0(q = 0, \omega_m = 0) = -T\sum_n \frac{\sum_{\mathbf{p}} G(\varepsilon_n, \mathbf{p}) - \sum_{\mathbf{p}} G(-\varepsilon_n, \mathbf{p})}{2i\varepsilon_n} = -T\sum_n \frac{\sum_{\mathbf{p}} G(\varepsilon_n, \mathbf{p})}{i\varepsilon_n}. \tag{19}$$

Performing now the standard summation over Matsubara frequencies we obtain:

$$\chi_0(q = 0, \omega_m = 0) = -\frac{1}{4\pi i}\int_{-\infty}^{\infty} d\varepsilon \frac{\sum_{\mathbf{p}} G^R(\varepsilon, \mathbf{p}) - \sum_{\mathbf{p}} G^A(\varepsilon, \mathbf{p})}{\varepsilon} th\frac{\varepsilon}{2T} = \int_{-\infty}^{\infty} d\varepsilon \frac{\tilde{N}(\varepsilon)}{2\varepsilon} th\frac{\varepsilon}{2T}, \tag{20}$$

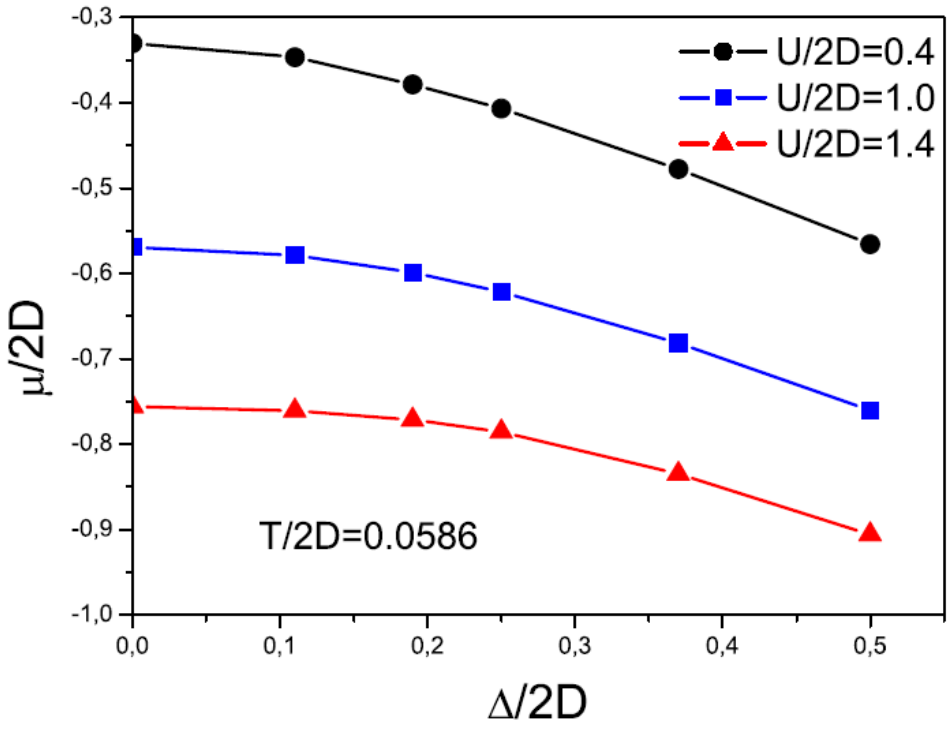
where $\tilde{N}(\varepsilon)$ is the density of states ($U = 0$) "dressed" by disorder scattering. In Eq. (20) the energy ε is reckoned from the chemical potential and if we reckon it from the center of conduction band we have to replace $\varepsilon \to \varepsilon - \mu$, so that the condition of Cooper instability (15) leads to the following equation for T_c:

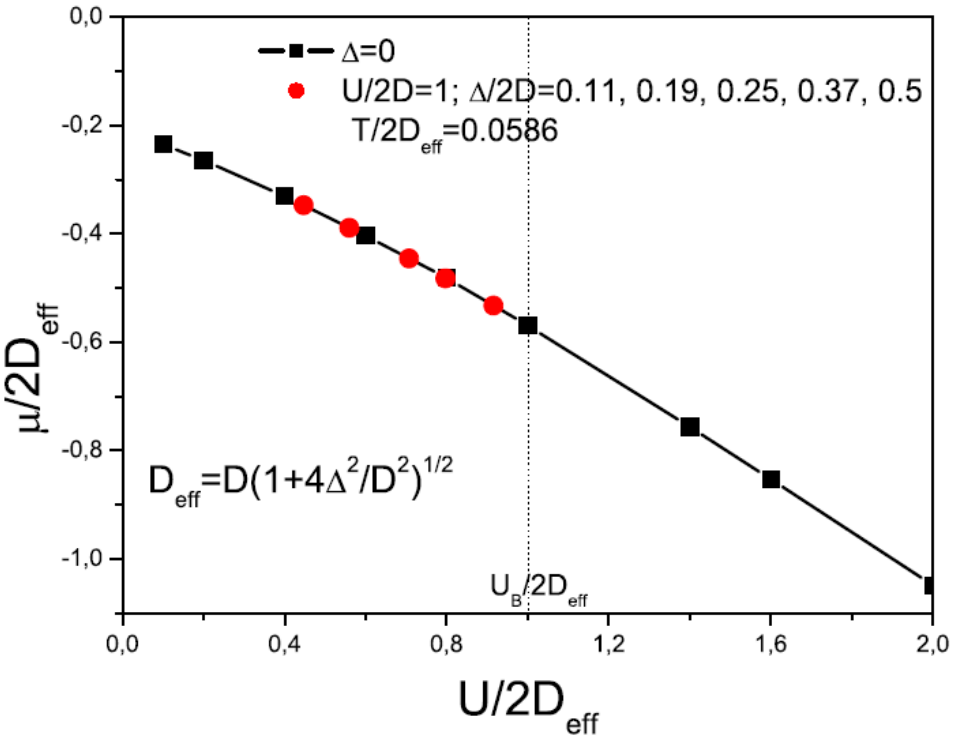
$$1 = \frac{U}{2}\int_{-\infty}^{\infty} d\varepsilon \tilde{N}_0(\varepsilon) \frac{th\frac{\varepsilon-\mu}{2T_c}}{\varepsilon - \mu} \tag{21}$$

where $\tilde{N}_0(\varepsilon)$ is again the density of states (calculated for $U = 0$) "dressed" by disorder scattering. At the same time, the chemical potential of the system at different values of U and Δ should be determined from DMFT+Σ calculations, i.e. from the standard equation for the number of electrons (band-filling), determined by Green's function

which allows us to find T_c for the wide range of model parameters, including the BCS-BEC crossover and strong coupling regions, as well as for different levels of disorder.

Chemical potential





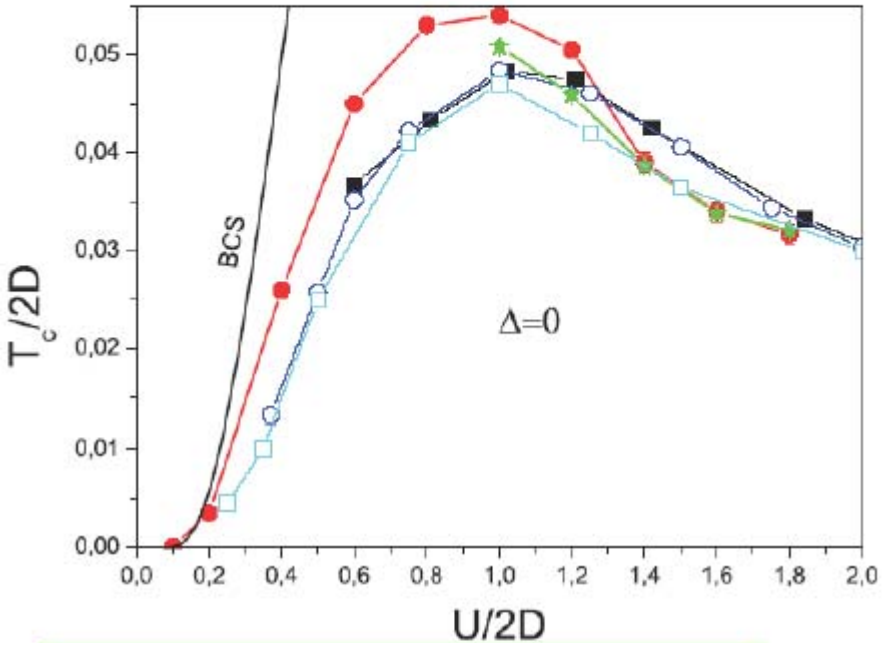
Bound state:

$$\frac{1}{U_B} = \int_{-D}^{D} d\xi \frac{N_0(\xi)}{2\xi + D}$$

$$U_B/2D = 1$$

$$\Delta \neq 0 \qquad D \to D_{eff}$$

Critical Temperature Tc



P. Nozieres and S. Schmitt-Rink, J. Low Temp. Phys. **59**, 195 (1985)

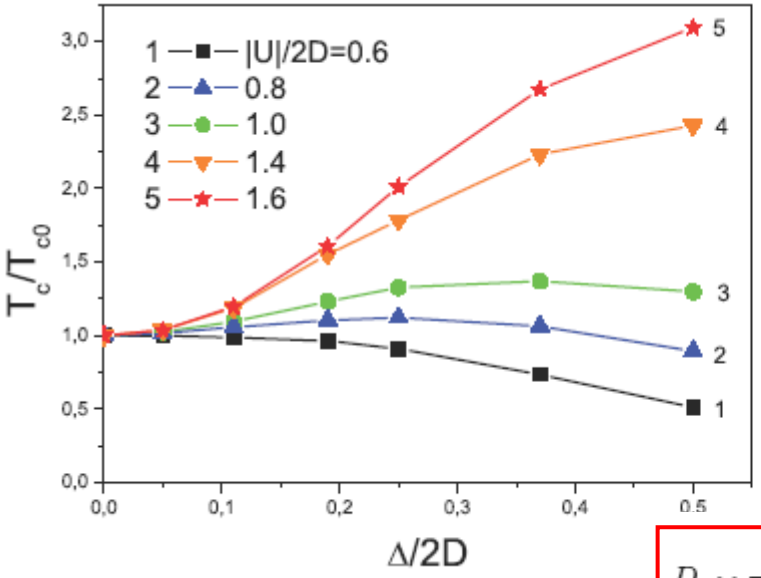
$$1 = \frac{|U|}{2}\int_{-D}^{D} d\varepsilon N_0(\varepsilon)\frac{th\frac{\varepsilon-\mu}{2T_c}}{\varepsilon-\mu}$$

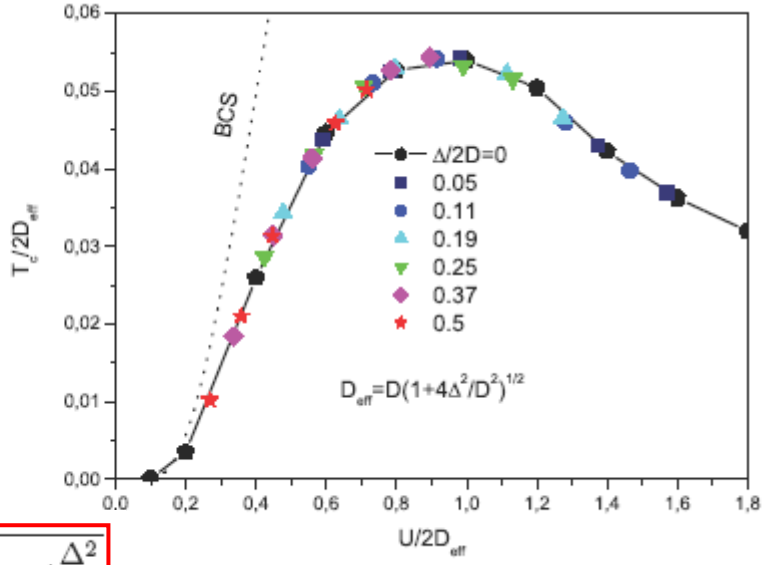
DMFT→μ

- M. Keller, W. Metzner, and U. Schollwock. Phys. Rev. Lett. **86**, 46124615 (2001)
- A. Toschi, P. Barone, M. Capone, and C. Castellani. New Journal of Physics 7, 7 (2005)
- A. Koga and P. Werner. Phys. Rev. A **84**, 023638 (2011)

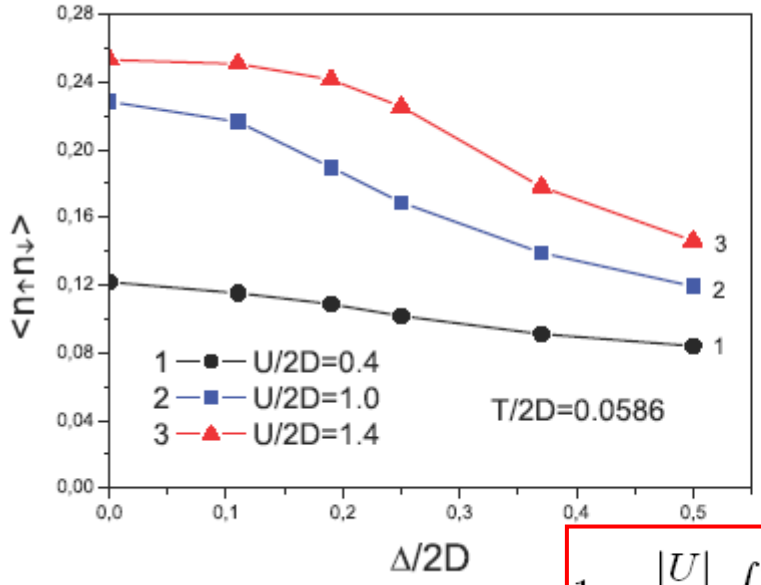
$T_c \sim t^2/|U|$ $\quad$ $|U| >> t \sim D$

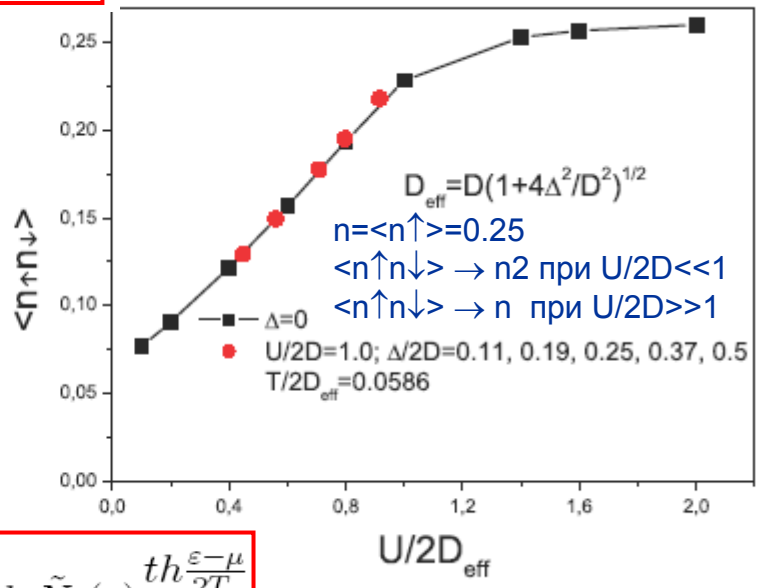
Tc and the Generalized Anderson Theorem





$$D_{eff} = D\sqrt{1+4\frac{\Delta^2}{D^2}}$$





$$1 = \frac{|U|}{2}\int_{-D_{eff}}^{D_{eff}} d\varepsilon \tilde{N}_0(\varepsilon)\frac{th\frac{\varepsilon-\mu}{2T_c}}{\varepsilon-\mu}$$

Tc and the Generalized Anderson Theorem

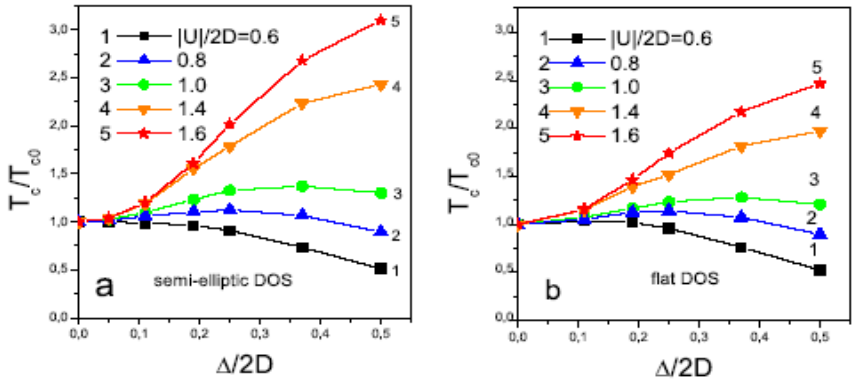


FIG. 3: Dependence of superconducting transition temperature on disorder for different values of Hubbard attraction U: (a) — semi – elliptic band; (b) — flat band.

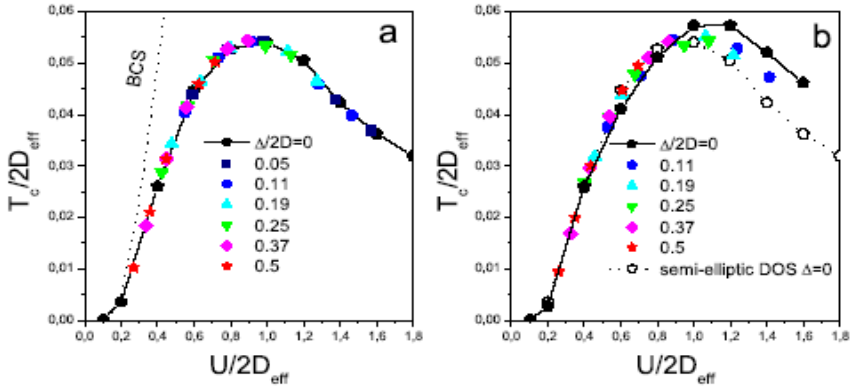
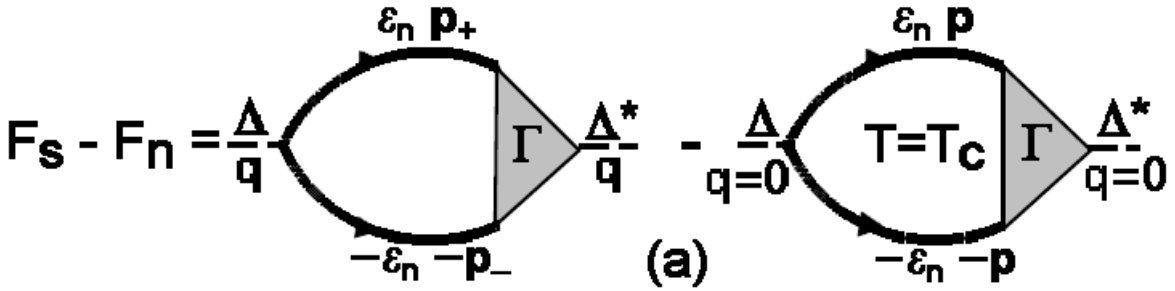
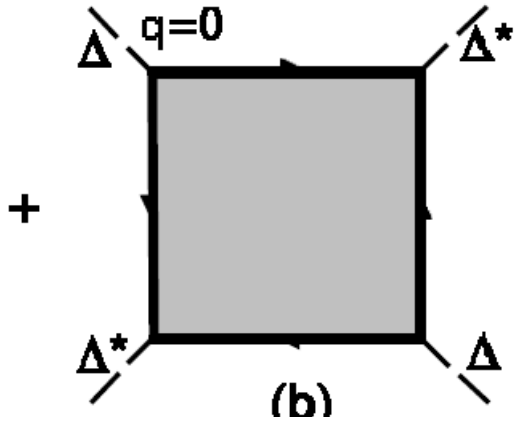


FIG. 4: Universal dependence of superconducting critical temperature on Hubbard attraction U for different disorder levels: (a) — semi – elliptic[tic band. Dashed curve represent BCS dependence in the absence of disorder. (b) — flat band. Dashed line represents similar dependence for semi – elliptic band for $\Delta = 0$.

Ginzburg-Landau Expansion



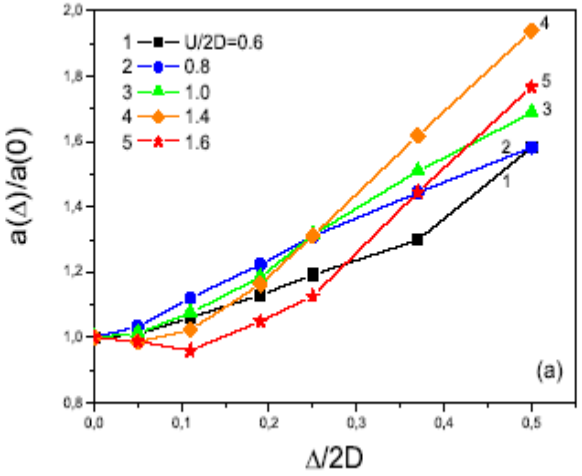


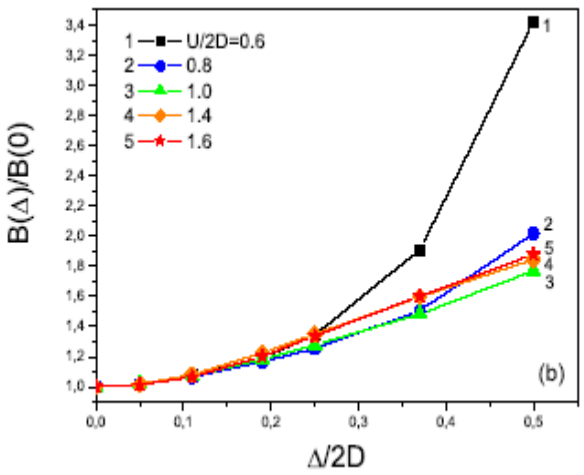
$$F_s - F_n = A|\Delta_q|^2 + q^2 C|\Delta_q|^2 + \frac{B}{2}|\Delta_q|^4$$

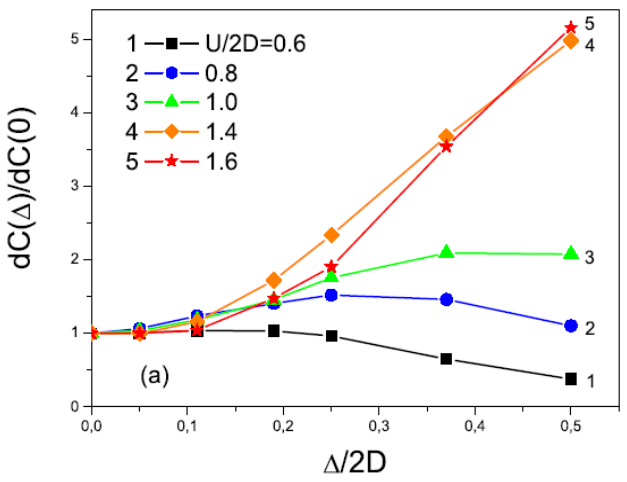
$$A(T) = \frac{1}{|U|} - \int_{-\infty}^{\infty} d\varepsilon \tilde{N}_0(\varepsilon) \frac{th\frac{\varepsilon-\mu}{2T}}{2(\varepsilon-\mu)} \quad B = \int_{-\infty}^{\infty} \frac{d\varepsilon}{4(\varepsilon-\mu)^3} \left(th\frac{\varepsilon-\mu}{2T_c} - \frac{(\varepsilon-\mu)/2T_c}{ch^2\frac{\varepsilon-\mu}{2T_c}} \right) \tilde{N}_0(\varepsilon)$$

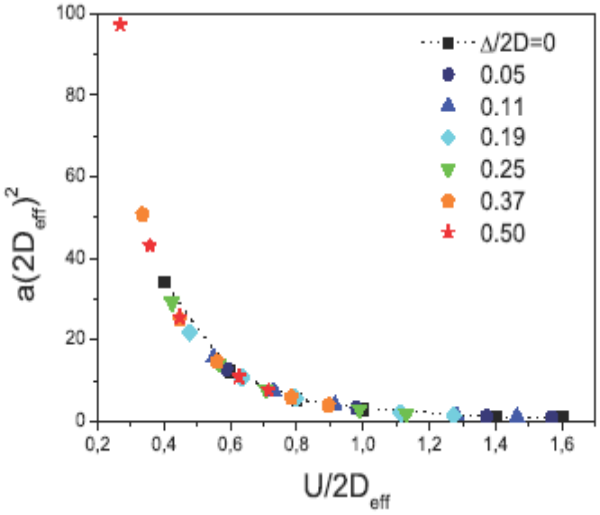
$$A(T) = a(T - T_c) \qquad C_s(T_c) - C_n(T_c) = T_c \frac{a^2}{B}$$

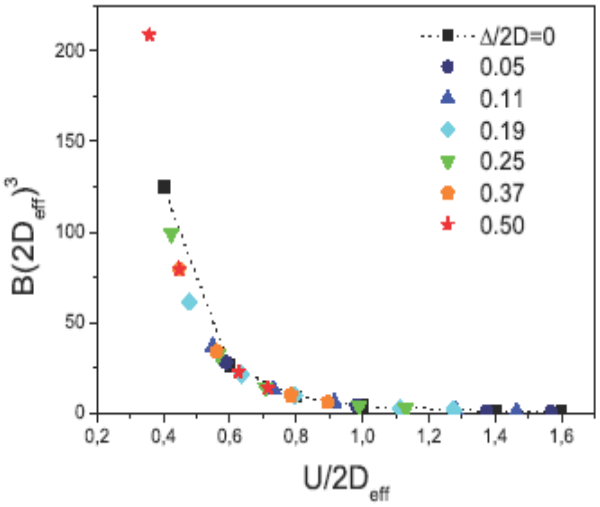
Ginzburg-Landau coefficients

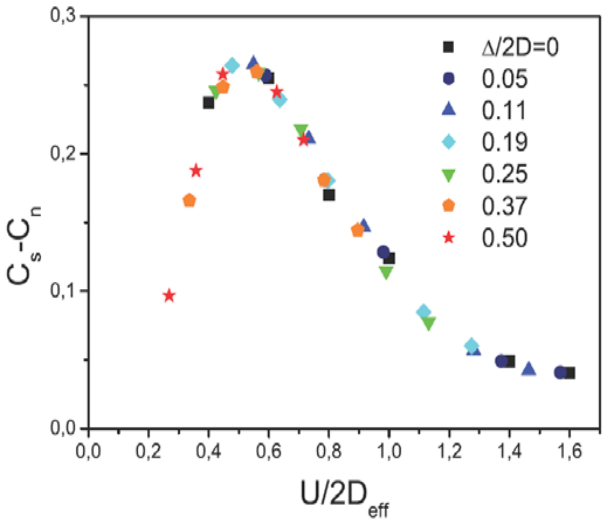












Выводы

- DMFT+Σ и модель Хаббарда с притяжением эффективно описывают БКШ-БЭК кроссовер, в том числе для системы с беспорядком
- T_c в неупорядоченной модели Хаббарда подчиняется обобщенной теореме Андерсона

Anderson Theorem

Nontrivial results concerning superconductivity in disordered systems were obtained very soon since the discovery of BCS—theory [Abrikosov A.A., Gorkov L.P. (1958); Abrikosov A.A., Gorkov L.P. (1959); Gorkov L.P. (1959); Anderson P.W. (1959)]. The concept of "dirty" superconductor described the experimentally very important case of the mean free path l short in comparison with superconducting coherence length $\xi_0 \sim \hbar v_F/T_c$, i.e. the case when:

$$\xi_0 \gg l \gg \hbar/p_F \tag{3.1}$$

Already in this case of not so strongly disordered (in the sense of closeness to metal—insulator transition) system Cooper pairing takes place not between electrons with opposite momenta and spins as in regular case, but between time—reversed exact eigenstates of electrons in disordered system [Anderson P.W. (1959); Gennes de P.G. (1966)]:

$$(\mathbf{p}_\uparrow, -\mathbf{p}_\downarrow) \Longrightarrow (\phi_\nu(\mathbf{r})_\uparrow, \phi_\nu^\star(\mathbf{r})_\downarrow) \tag{3.2}$$

Then the linearized gap equation determining T_c takes the form:

$$\Delta(\mathbf{r}) = gT \int d\mathbf{r}' \sum_{\varepsilon_n} K(\mathbf{r}\mathbf{r}'\varepsilon_n)\Delta(\mathbf{r}') \tag{3.7}$$

where the kernel:

$$K(\mathbf{r}\mathbf{r}'\varepsilon_n) = G_\uparrow(\mathbf{r}\mathbf{r}'\varepsilon_n)G^\star_\downarrow(\mathbf{r}'\mathbf{r}\varepsilon_n) \tag{3.8}$$

is formed by exact one—electron Green's functions of a normal metal. Now we can use an exact eigenstate representation for an electron in a random field of a disordered system to write (Cf. Eq. (A.13)):

$$G_\uparrow(\mathbf{r}\mathbf{r}'\varepsilon_n) = \sum_\nu \frac{\phi_{\nu\uparrow}(\mathbf{r})\phi^\star_{\nu\uparrow}(\mathbf{r}')}{i\varepsilon_n - \varepsilon_\nu} \tag{3.9}$$

where ε_ν are exact energy levels of an electron in disordered system. Then

$$K(\mathbf{r}\mathbf{r}'\varepsilon_n) = Tg \sum_{\mu\nu} \frac{\phi_{\nu\uparrow}(\mathbf{r})\phi^\star_{\nu\uparrow}(\mathbf{r}')\phi^\star_{\mu\downarrow}(\mathbf{r}')\phi_{\mu\downarrow}(\mathbf{r})}{(i\varepsilon_n - \epsilon_\nu)(-i\varepsilon_n + \epsilon_\mu)} \tag{3.10}$$

In the following for brevity we shall drop spin variables always assuming singlet pairing. In case of a system with time—reversal invariance (i.e. in the absence of an external magnetic field, magnetic impurities etc.) Eq. (3.10) can be rewritten as:

$$K(\mathbf{r}\mathbf{r}'\varepsilon_n) = G(\mathbf{r}\mathbf{r}'\varepsilon_n)G(\mathbf{r}'\mathbf{r} - \varepsilon_n) = \sum_{\mu\nu} \frac{\phi_\nu(\mathbf{r})\phi^\star_\nu(\mathbf{r}')\phi_\mu(\mathbf{r}')\phi^\star_\mu(\mathbf{r})}{(i\varepsilon_n - \varepsilon_\nu)(-i\varepsilon_n - \varepsilon_\mu)} \tag{3.11}$$

Averaging over disorder we get:

$$< \Delta(\mathbf{r}) >= gT \int d\mathbf{r}' \sum_{\varepsilon_n} < K(\mathbf{r}\mathbf{r}'\varepsilon_n)\Delta(\mathbf{r}') > \tag{3.12}$$

$$< \Delta(\mathbf{r}) >= gT \int d\mathbf{r}' \sum_{\varepsilon_n} K(\mathbf{r}-\mathbf{r}'\varepsilon_n) < \Delta(\mathbf{r}') > \qquad (3.13)$$

where the averaged kernel in case of time—invariance is given by:

$$K(\mathbf{r}-\mathbf{r}'\varepsilon_n) = K^{\star}(\mathbf{r}-\mathbf{r}'\varepsilon_n) =< K(\mathbf{r}\mathbf{r}'\varepsilon_n) >=$$
$$=< \sum_{\mu\nu} \frac{\phi_\nu(\mathbf{r})\phi^{\star}_\mu(\mathbf{r})\phi_\mu(\mathbf{r}')\phi^{\star}_\nu(\mathbf{r}')}{(i\varepsilon_n-\varepsilon_\nu)(-i\varepsilon_n-\varepsilon_\mu)} >=$$
$$= \int_{-\infty}^{\infty} dE N(E) \int_{-\infty}^{\infty} d\omega \frac{\ll \rho_E(\mathbf{r})\rho_{E+\omega}(\mathbf{r}') \gg^F}{(i\varepsilon_n+E)(E+\omega-i\varepsilon_n)} \qquad (3.14)$$

where we have introduced Gorkov—Berezinskii spectral density [Berezinskii V.L., Gorkov L.P. (1979)] (Cf. Eq. (A2)):

$$\ll \rho_E(\mathbf{r})\rho_{E+\omega}(\mathbf{r}') \gg^F =$$
$$\frac{1}{N(E)} < \sum_{\mu\nu} \phi^{\star}_\nu(\mathbf{r})\phi_\mu(\mathbf{r})\phi^{\star}_\mu(\mathbf{r}')\phi_\nu(\mathbf{r}')\delta(E-\varepsilon_\nu)\delta(E+\omega-\varepsilon_{\nu'}) > \qquad (3.15)$$

Here $N(E)$ is an exact electron density of states per *one spin direction* as it always appears in superconductivity theory (above, while discussing localization we almost always used density of states for both spin directions).

Usually the decoupling procedure used in Eq. (3.12) to reduce it to Eq. (3.13) is justified by the assumption that the averaging of $\Delta(\mathbf{r})$ and of Green's functions in Eq. (3.12) forming the kernel can be performed independently because of essentially different spatial scales [Gorkov L.P. (1959)]: $\Delta(\mathbf{r})$ changes at a scale of the order of coherence length (Cooper pair size) ξ, while $G(\mathbf{r}\mathbf{r}'\varepsilon_n)$ are oscillating on the scale of interatomic distance $a \sim \hbar/p_F$, and we always have $\xi \gg a$. Actually it is clear that this decoupling is valid only if the order—parameter is *self—averaging* (i.e. in fact nonrandom) quantity: $\Delta(\mathbf{r}) =< \Delta(\mathbf{r}) >$, $< \Delta^2(\mathbf{r}) >=< \Delta(\mathbf{r}) >^2$.

If we look for the solution of Eq. (3.13) $\Delta(\mathbf{r}) = const$ (homogeneous gap), we immediately obtain the following equation for superconducting transition temperature T_c:

$$1 = gT_c \int d\mathbf{r} \sum_{\varepsilon_n} K(\mathbf{r} - \mathbf{r}' \varepsilon_n) =$$

$$= gT_c \int d\mathbf{r} \sum_{\varepsilon_n} \int_{-\infty}^{\infty} dE N(E) \int_{-\infty}^{\infty} d\omega \frac{\ll \rho_E(\mathbf{r}) \rho_{E+\omega}(\mathbf{r}') \gg^F}{(E + i\varepsilon_n)(E + \omega - i\varepsilon_n)} \qquad (3.16)$$

Using the general sum—rule given in Eq. (A.5) [Berezinskii V.L., Gorkov L.P. (1979)]:

$$\int d\mathbf{r} \ll \rho_E(\mathbf{r}) \rho_{E+\omega}(\mathbf{r}') \gg^F = \delta(\omega) \qquad (3.17)$$

we immediately reduce Eq. (3.16) to a standard BCS form:

$$1 = gT_c \int_{-\infty}^{\infty} dE N(E) \sum_{\varepsilon_n} \frac{1}{E^2 + \varepsilon_n^2} = g \int_0^{<\omega>} dE N(E) \frac{1}{E} th \frac{E}{2T_c} \qquad (3.18)$$

where we introduced the usual cut—off at $E \sim 2 < \omega >$. Note that $N(E)$ here is an exact one-particle density of states (per one spin direction) in a normal state of a disordered system. From Eq. (3.18) we get the usual result:

$$T_c = \frac{2\gamma}{\pi} < \omega > exp\left(-\frac{1}{\lambda_p}\right) \qquad (3.19)$$

where $\lambda_p = gN(E_F)$ is dimensionless pairing constant, $ln\gamma = C = 0.577...$ is Euler constant. This is the notorious Anderson theorem: in the absence of scattering processes breaking time—reversal invariance disorder influence T_c only through the possible changes of the density of states $N(E_F)$ under disordering (which are usually relatively small).

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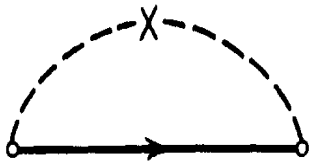
ЭФФЕКТЫ РАЗУПОРЯДОЧЕНИЯ В СВЕРХПРОВОДНИКАХ С АНИЗОТРОПНЫМ СПАРИВАНИЕМ: ОТ КУПЕРОВСКИХ ПАР К КОМПАКТНЫМ БОЗОНАМ

М.В.Садовский[1)], *А.И.Посаженникова*



В случае системы с примесями точка бозе-конденсации определяется следующим уравнением [8]:

$$\mu_p - \sum(0) = 0, \tag{3}$$

где μ_p - химический потенциал пар, а $\sum(0)$ - предел нулевой частоты собственно-энергетической части бозона в поле примесей, которая в случае слабого рассеяния определяется диаграммой, показанной на рис.1:

$$\sum(\epsilon_n) = n_{imp} v^2 \int \frac{d^3\mathrm{p}}{(2\pi)^3} \frac{1}{i\epsilon_n - \mathrm{p}^2/2m^* + \mu_p}, \tag{4}$$

где $\epsilon_n = 2\pi nT$ - четная мацубаровская частота, $m^* = 2m$ - масса пары, и мы предполагаем $T > T_c$. В дальнейшем мы ограничимся рассмотрением трехмерных систем. Непосредственные вычисления дают

$$\sum(0) = \mathrm{Re}\tilde{\Sigma}(0) + E_{0c}, \tag{5}$$

где $E_{0c} = -(m^*/\pi^2) n_{imp} v^2 p_0$ - сдвиг края зоны, вызываемый рассеянием на примесях [9] (p_0 - параметр обрезания в импульсном пространстве порядка обратной постоянной решетки a^{-1}), а

$$\mathrm{Re}\tilde{\Sigma}(0) = \frac{1}{\sqrt{2}\pi} n_{imp} v^2 m^{*3/2} \sqrt{|\mu_p|}. \tag{6}$$

Величина E_{0c} приводит к простой перенормировке химического потенциала: $\tilde{\mu} = \mu_p - E_{0c}$, так что в перенормированном виде уравнение (3) сводится к

$$\tilde{\mu}\left(1 - \frac{1}{\sqrt{2|\tilde{\mu}|}\pi} n_{imp} v^2 m^{*3/2} \mathrm{sign}\tilde{\mu}\right) = 0 \tag{7}$$

с одним существенным для нас ($\tilde{\mu} < 0$ для бозонов при $T > T_c$) корнем: $\tilde{\mu} = 0$, то есть $\mu_p - E_{0c} = 0$. Соответственно, температура бозе-конденсации в примесной системе определяется стандартным уравнением:

$$\frac{n}{2} = g \int_{-\infty}^{\infty} d\epsilon N(\epsilon) \frac{1}{e^{\epsilon/T_c} - 1}, \tag{8}$$

где $g = 2s + 1$ (для бозонов со спином s), $N(\epsilon)$ - усредненная по примесям плотность состояний, которая в простейшем приближении (4) сводится к $N(E - E_{0c})$ - плотности состояний свободных частиц с энергией ϵ, отсчитываемой от сдвинутого края зоны. Очевидно, что отсюда следует обычное выражение для T_c [10]:

$$T_c = \frac{3.31}{g^{2/3}} \frac{(n/2)^{2/3}}{m^*}, \tag{9}$$

которое *не зависит от беспорядка*. Возможное влияние беспорядка может быть связано лишь с экспоненциально малым "лифшицевским хвостом" в плотности состояний в уравнении (8), связанным с локализацией [11], который не возникает в нашем простейшем приближении (4). Таким образом, мы приходим к выводу, что в приближении очень сильного спаривательного взаимодействия (сверхпроводимость компактных пар) T_c практически не зависит от беспорядка для *любого* значения спина куперовской пары, то есть для пар s-типа, d-типа и так далее.